OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4729

Mechanics 2

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use 9.8 m s^{-2}.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

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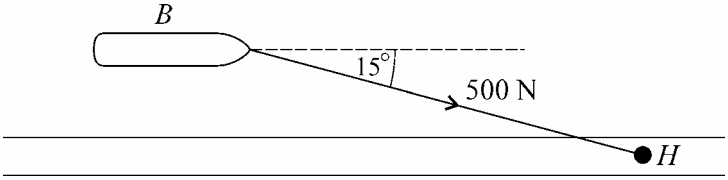
1



A barge B is pulled along a canal by a horse H, which is on the tow-path. The barge and the horse move in parallel straight lines and the tow-rope makes a constant angle of 15° with the direction of motion (see diagram). The tow-rope remains taut and horizontal, and has a constant tension of 500 N.

(i) Find the work done on the barge by the tow-rope, as the barge travels a distance of 400 m. [3]

The barge moves at a constant speed and takes 10 minutes to travel the 400 m.

(ii) Find the power applied to the barge. [2]

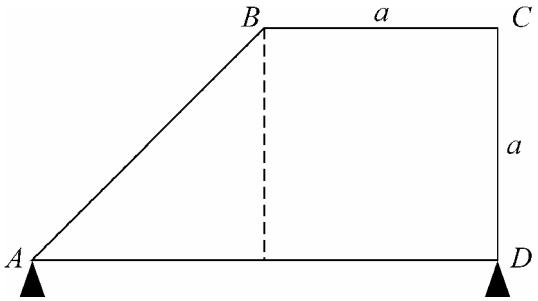
2 A uniform circular cylinder, of radius 6 cm and height 15 cm, is in equilibrium on a fixed inclined plane with one of its ends in contact with the plane.

(i) Given that the cylinder is on the point of toppling, find the angle the plane makes with the horizontal. [3]

The cylinder is now placed on a horizontal board with one of its ends in contact with the board. The board is then tilted so that the angle it makes with the horizontal gradually increases.

(ii) Given that the coefficient of friction between the cylinder and the board is $\frac{3}{4}$, determine whether or not the cylinder will slide before it topples, justifying your answer. [4]

3

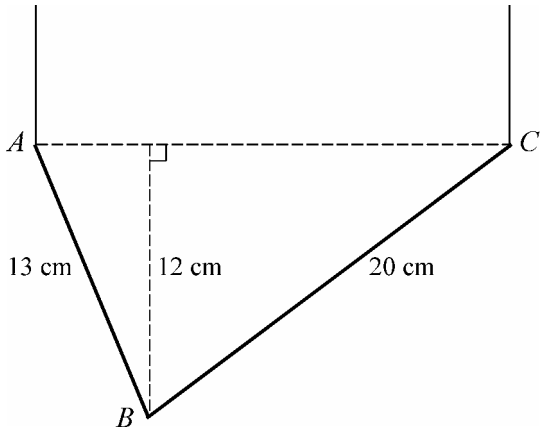


A uniform lamina $ABCD$ has the shape of a square of side a adjoining a right-angled isosceles triangle whose equal sides are also of length a. The weight of the lamina is W. The lamina rests, in a vertical plane, on smooth supports at A and D, with AD horizontal (see diagram).

(i) Show that the centre of mass of the lamina is at a horizontal distance of $\frac{11}{9}a$ from A. [4]

(ii) Find, in terms of W, the magnitudes of the forces on the supports at A and D. [4]

4



A rigid body ABC consists of two uniform rods AB and BC, rigidly joined at B. The lengths of AB and BC are 13 cm and 20 cm respectively, and their weights are 13 N and 20 N respectively. The distance of B from AC is 12 cm. The body hangs in equilibrium, with AC horizontal, from two vertical strings attached at A and C. Find the tension in each string. [8]

5 A cyclist and his machine have a combined mass of 80 kg. The cyclist ascends a straight hill AB of constant slope, starting from rest at A and reaching a speed of 5 m s^{-1} at B. The level of B is 4 m above the level of A.

(i) Find the gain in kinetic energy and the gain in gravitational potential energy of the cyclist and his machine. [3]

During the ascent the resistance to motion is constant and has magnitude 70 N.

(ii) Given that the work done by the cyclist in ascending the hill is 8000 J, find the distance AB. [3]

At B the cyclist is working at 720 watts and starts to move in a straight line along horizontal ground. The resistance to motion has the same magnitude of 70 N as before.

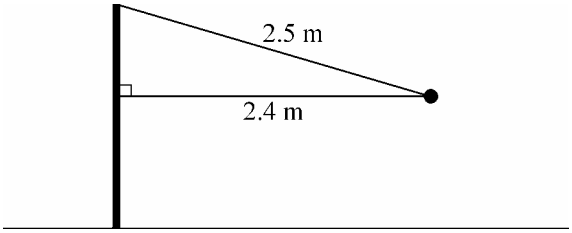
(iii) Find the acceleration with which the cyclist starts to move horizontally. [4]

6 An athlete 'puts the shot' with an initial speed of 19 m s^{-1} at an angle of 11° above the horizontal. At the instant of release the shot is 1.53 m above the horizontal ground. By treating the shot as a particle and ignoring air resistance, find

(i) the maximum height, above the ground, reached by the shot, [4]

(ii) the horizontal distance the shot has travelled when it hits the ground. [6]

7



A ball of mass 0.08 kg is attached by two strings to a fixed vertical post. The strings have lengths 2.5 m and 2.4 m, as shown in the diagram. The ball moves in a horizontal circle, of radius 2.4 m, with constant speed v m s^{-1}. Each string is taut and the lower string is horizontal. The modelling assumptions made are that both strings are light and inextensible, and that there is no air resistance.

(i) Find the tension in each string when $v = 10.5$. [7]

(ii) Find the least value of v for which the lower string is taut. [4]

8 Two uniform smooth spheres, A and B, have the same radius. The mass of A is 0.24 kg and the mass of B is m kg. Sphere A is travelling in a straight line on a horizontal table, with speed 8 m s^{-1}, when it collides directly with sphere B, which is at rest. As a result of the collision, sphere A continues in the same direction with a speed of 6 m s^{-1}.

(i) Find the magnitude of the impulse exerted by A on B. [3]

(ii) Show that $m \leqslant 0.08$. [3]

It is given that $m = 0.06$.

(iii) Find the coefficient of restitution between A and B. [3]

On another occasion A and B are travelling towards each other, each with speed 4 m s^{-1}, when they collide directly.

(iv) Find the speeds of A and B immediately after the collision. [4]

1	**(i)**	Work done is $500\cos 15° \times 400 \approx 193\,000$ J	M1	For attempt to use Force × distance
			A1	For correct unsimplified product
			A1 **3**	For correct answer 193 000
	(ii)	Power applied is $\frac{193\,185}{600} \approx 322$ W	M1	For relevant use of $\frac{\text{work}}{\text{time}}$ or force × velocity
			A1 **2**	For correct answer 322
			5	
2	**(i)**	CM is vertically above lowest point of base	B1	For stating or implying correct geometry
		Hence $\tan\alpha = \frac{6}{7.5} \Rightarrow \alpha = 38.7°$	M1	For appropriate trig calculation
			A1 **3**	For correct answer 38.7
	(ii)	Cylinder slides when $\tan\theta = \frac{3}{4}$	B1	For stating or implying limiting friction case
		But $\frac{3}{4} < 0.8$, so $\theta < \alpha$	M1	For comparing $\tan\alpha$ to $\tan\theta$, or equivalent
			A1	For correct comparison of the angles
		Hence it slides first (at inclination $36.9°$)	A1 **4**	For correct conclusion of sliding first
			7	
3	**(i)**	CG of triangle is $\frac{2}{3}a$ horizontally from A	B1	
		Moments: $\frac{1}{3}W \times \frac{2}{3}a + \frac{2}{3}W \times \frac{3}{2}a = W \times \bar{x}$	M1	For equating moments about A, or equivalent
			A1	For a correct unsimplified equation
		Hence $\bar{x} = \frac{11}{9}a$	A1 **4**	Given answer correctly shown
	(ii)	$R_A \times 2a = W \times \frac{7}{9}a \Rightarrow R_A = \frac{7}{18}W$	M1	For one moments equation
			A1	For one correct answer
		$R_A + R_D = W \Rightarrow R_D = \frac{11}{18}W$	M1	For resolving, or a second moments equation
			A1√ **4**	For a second correct answer
			8	
4		Horiz distances of B from A and C are 5 cm and 16 cm	M1	For appropriate use of Pythagoras
			A1	For both distances correct
		$21T_A = 13 \times 18.5 + 20 \times 8$	M1	For any moments equation for the system
			A1√	For any one relevant term correct
			A1√	For a completely correct equation
		$T_A + T_C = 33$	M1	For resolving, or using another moments eqn
		Hence $T_A = 19.1$ N and $T_C = 13.9$ N	A1	For correct answer 19.1
			A1 **8**	For correct answer 13.9
			8	
5	**(i)**	Gain in KE is $\frac{1}{2} \times 80 \times 5^2 = 1000$ J	M1	For use of formula $\frac{1}{2}mv^2$
		Gain in PE is $80 \times 9.8 \times 4 = 3136$ J	M1	For use of formula mgh
			A1 **3**	For both answers 1000 and 3136 correct
	(ii)	$8000 = 1000 + 3136 + 70d$	M1	For equating work done to energy change
			M1	For relevant use of force × distance
		Hence distance AB is 55.2 m	A1 **3**	For correct answer 55.2
	(iii)	$\frac{720}{5} - 70 = 80a$	B1	For driving force $\frac{720}{5}$
			M1	For use of Newton II with 3-term equation
			A1	For a completely correct equation
		Hence acceleration is 0.925 m s^{-2}	A1 **4**	For correct answer 0.925
			10	

6	**(i)**	$0 = (19\sin 11°)^2 - 2gh$	M1	For use of relevant const acc equation for h
			B1	For correct vertical component $19\sin 11°$
		Hence max height is $\dfrac{(19\sin 11°)^2}{19.6} + 1.53 = 2.20$ m	A1	For correct expression for h (≈ 0.67)
			A1 **4**	For correct answer 2.20
	(ii)	*EITHER*: Time to top point is $\dfrac{19\sin 11°}{g} \approx 0.3699$	M1	For use of relevant const acc equation for t_{up}
		Time to fall is $\sqrt{\dfrac{2\times 2.20}{9.8}} \approx 0.6701$	M1	For use of relevant const acc eqn for t_{down}
			A1	For a correct expression for t_{down}
		Total time of flight is 1.04	A1	For correct value (or expression)
		Horiz dist is $19\cos 11° \times 1.04 \approx 19.4$ m	M1	For any use of $x = (19\cos 11°)t$
			A1	For correct answer 19.4
				[Alternative approaches for the first four marks are equally acceptable; e.g. the use of $s = ut - \frac{1}{2}gt^2$ to find $t = 1.04$]
		OR: $-1.53 = x\tan 11° - \dfrac{gx^2}{2\times(19\cos 11°)^2}$	M1	For relevant use of trajectory equation
			B1	For $y = -1.53$ correctly substituted
			A1	For completely correct equation for x
		Hence $x = 19.4$	M1	For attempt to solve relevant quadratic
			A2 **6**	For correct answer 19.4
			10	
7	**(i)**	$T_1 \times \frac{7}{25} = 0.08g$	M1	For resolving vertically
			B1	For $\frac{7}{25}$ or $\sin 16.3°$ or equivalent
		Hence tension in upper string is 2.8 N	A1	For correct value 2.8
		$T_1 \times \frac{24}{25} + T_2 = 0.08 \times \dfrac{10.5^2}{2.4}$	M1	For correct use of Newton II horizontally
			B1	For any use of $\dfrac{10.5^2}{2.4}$, or equivalent
			A1	For correct horizontal equation
		Hence tension in horizontal string is 0.987 N	A1 **7**	For correct value 0.987
	(ii)	$2.8 \times \dfrac{2.4}{2.5} = 0.08 \times \dfrac{v^2}{2.4}$	M1	For new horizontal equation with $T_2 = 0$
			A1√	For correct equation for v
		Hence $v = 8.98$	M1	For solving for v correctly
			A1 **4**	For correct value 8.98
			11	

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8	**(i)**	Change of momentum of A is 0.24×2	M1		For considering momentum of A
			A1		For correct expression for change in mom
		Hence magnitude of impulse is 0.48 N s	A1	**3**	For correct answer 0.48
	(ii)	$mv_B = 0.48$	M1		For considering momentum of B
		$v_B \geqslant 6$	M1		For using the inequality $v_B \geqslant v_A$
		Hence $m \leqslant \dfrac{0.48}{6} = 0.08$	A1	**3**	For showing given answer correctly
	(iii)	$m = 0.06 \Rightarrow v_B = 8$	B1		For correct speed of B
		Hence $8 - 6 = e(8 - 0)$	M1		For correct use of Newton's law
		i.e. $e = \frac{1}{4}$	A1	**3**	For correct answer $\frac{1}{4}$ or equivalent
	(iv)	$0.24\times 4 - 0.06\times 4 = 0.24a + 0.06b$	B1		For a correct momentum equation
		$b - a = \frac{1}{4}(4 + 4)$	B1√		For a correct restitution equation
		Hence speeds of A and B are $2\ \text{m s}^{-1}$ and $4\ \text{m s}^{-1}$	M1		For solution of relevant simultaneous equns
			A1	**4**	For both answers correct
				13	